

Eliminating the chiral anomaly *via* symplectic embedding approach

A. C. R. Mendes^{a*}, C. Neves^{b†}, W. Oliveira^{b‡}

^a*Universidade Federal de Viçosa,*

Campus Rio Paranaíba,

38810-000, Rio Paranaíba, MG

^b*Departamento de Física, ICE,*

Universidade Federal de Juiz de Fora,

36036-330, Juiz de Fora, MG, Brazil

December 15, 2009

The quantization of the chiral Schwinger model (χQED_2) with one-parameter class Faddeevian regularization is hampered by the chiral anomaly, *i.e.*, the Gauss law commutator exhibits Faddeev's anomaly. To overcome this kind of problem, we propose to eliminate this anomaly by embedding the theory through a new gauge-invariant formalism based on the enlargement of the phase space with the introduction of Wess-Zumino(WZ) fields and the symplectic approach[1, 2]. This process opens up a possibility to formulate different, but dynamically equivalent, gauge invariant versions for the model and also gives a geometrical interpretation to the arbitrariness presents on the BFFT and iterative conversion methods. Further, we observe that the elimination of the chiral anomaly imposes a condition on the chiral parameters present on the original model and on the WZ sector.

* E-mail: albert@ufv.br

† E-mail: cneves@fisica.ufjf.br

‡ E-mail: wilson@fisica.ufjf.br

I. INTRODUCTION

It has been shown over the last decade that anomalous gauge theories in two dimensions can be consistently and unitarily quantized for both Abelian[3, 4, 5] and non-Abelian[6, 7] cases. In this scenario, the two dimensional model that has been extensively studied is the chiral Schwinger model(CSM)[3]. In these early works were considered to study an effective action with a simple one-parameter class of regularization. The consequences of these constraint structures are that the $a > 1$ class presents, besides the massless excitation also a massive scalar excitation ($m^2 = \frac{e^2 a^2}{a-1}$) that is not found on the $a = 1$ class. In order to elucidate the physical spectrum, this model was analyzed by a variety of methods[4, 8, 9, 10, 11]. In Refs.[9, 10, 11] a gauge invariant formulation of the model with Wess-Zumino(WZ) fields was studied, as suggested in Ref.[12].

Despite this spate of interest, Mitra [13] proposed a new and surprising bosonized action for the Schwinger model with a new regularization prescription, that is different from those involved in the class of models studied earlier. In this paper he proposed a new (Faddeevian) regularization class, incarnated by a conveniently mass-like term which leads to a canonical description with three second class constraints, in the Dirac's context. In this paper, it has been shown that the Gauss law commutator exhibits Faddeev's anomaly. This model has an advantage because the Faddeevian mechanism[14], which is related to the anomalous Gauss law algebra in the anomalous gauge theories, works well. Recall that in [3] and [4], the Hamiltonian framework was structured in terms of two classes with two and four second class constraints respectively. Mitra's work brings a clear interpretation about the reason leading the bosonization ambiguity to fit into 3 distinct classes, classified according to the number of constraints present in the model. Recently, an extension of the Ref.[13] to an one-parameter class of solutions was proposed by one of us in [15] in order to study the restrictions posed by the soldering formalism[16] over this new regularization class. In this context, the gauge field becomes massive once again and its dependence on the ambiguity parameter was shown to be identical to that in Ref.[3], while the massless sector however is more constrained than its counterpart in [3], which corroborates the Mitra's finding outs[13].

In the present paper, the chiral Schwinger model (χQED_2) with one-parameter class Faddeevian regularization will be reformulated as a gauge invariant theory in order to eliminate the chiral anomaly that obstructed the quantization procedure. It is done just enlarging the phase space with the introduction of WZ fields. To this end, we chose to use a new gauge-invariant formalism, that is an extension of the symplectic gauge-invariant formalism[1, 2]. This formalism is developed in a way to handle constrained systems, called the symplectic framework [17, 18]. The basic object

behind this formalism is the symplectic matrix: if this matrix is singular, the model presents a symmetry. This is achieved introducing an arbitrary function (G), written in terms of the original and WZ variables, into the zeroth-iterative first-order Lagrangian.

In section II, we introduce the symplectic embedding formalism in order to settle the notation and familiarize the reader with the fundamentals of the formalism. In section III, the chiral Schwinger model with one-parameter class Faddeevian regularization will be introduced following so closely the original presentation[15]. Furthermore, this model will be investigated through the symplectic method[17, 18], displaying its chiral noninvariant nature, and the Dirac's brackets among the phase-space coordinates will be also computed.

In section IV, the chiral Schwinger model with one-parameter Faddeevian regularization will be reformulated as a gauge invariant theory with the introduction of WZ field *via* the new symplectic gauge-invariant formalism, presented in the Section II. An immediate consequence produced by this process is the generation of the infinitesimal gauge transformation that keep the Hamiltonian invariant and the elimination of the chiral anomaly. In this way, we have several versions for this model, which are described by different gauge invariant Hamiltonians, but all dynamically equivalent.

The last section is reserved to discuss the physical meaning of our findings together with our final comments and conclusions.

II. GENERAL FORMALISM

In this section, we describe the alternative embedding technique that changes the second class nature of a constrained system to the first one. This technique follows the Faddeev and Shatashivilli idea[12] and is based on a contemporary framework that handles constrained models, namely, the symplectic formalism[17, 18].

In order to systematize the symplectic embedding formalism, we consider a general noninvariant mechanical model whose dynamics is governed by a Lagrangian $\mathcal{L}(a_i, \dot{a}_i, t)$ (with $i = 1, 2, \dots, N$), where a_i and \dot{a}_i are the space and velocities variables, respectively. Notice that this model does not result in the loss of generality or physical content. Following the symplectic method the zeroth-iterative first-order Lagrangian is written as

$$\mathcal{L}^{(0)} = A_{\alpha}^{(0)} \dot{\xi}^{(0)\alpha} - V^{(0)}, \quad (1)$$

where the symplectic variables are

$$\xi^{(0)\alpha} = \begin{cases} a_i, & \text{with } \alpha = 1, 2, \dots, N \\ p_i, & \text{with } \alpha = N + 1, N + 2, \dots, 2N. \end{cases} \quad (2)$$

with $A_\alpha^{(0)}$ are the one-form canonical momenta and $V^{(0)}$ is the symplectic potential. The symplectic tensor is given by

$$f_{\alpha\beta}^{(0)} = \frac{\partial A_\beta^{(0)}}{\partial \xi^{(0)\alpha}} - \frac{\partial A_\alpha^{(0)}}{\partial \xi^{(0)\beta}}. \quad (3)$$

If this symplectic matrix is singular, it has a zero-mode ($\nu^{(0)}$) which can generate a new constraint when contracted with the gradient of symplectic potential,

$$\Omega^{(0)} = \nu^{(0)\alpha} \frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}}. \quad (4)$$

This constraint is introduced into the zeroth-iterative Lagrangian, Eq.(1), through a Lagrange multiplier η , generating the next one

$$\begin{aligned} \mathcal{L}^{(1)} &= A_\alpha^{(0)} \dot{\xi}^{(0)\alpha} + \dot{\eta} \Omega^{(0)} - V^{(0)}, \\ &= A_\gamma^{(1)} \dot{\xi}^{(1)\gamma} - V^{(1)}, \end{aligned} \quad (5)$$

with $\gamma = 1, 2, \dots, (2N + 1)$ and

$$\begin{aligned} V^{(1)} &= V^{(0)}|_{\Omega^{(0)}=0}, \\ \xi^{(1)\gamma} &= (\xi^{(0)\alpha}, \eta), \\ A_\gamma^{(1)} &= (A_\alpha^{(0)}, \Omega^{(0)}). \end{aligned} \quad (6)$$

As a consequence, the first-iterative symplectic tensor is computed as

$$f_{\gamma\beta}^{(1)} = \frac{\partial A_\beta^{(1)}}{\partial \xi^{(1)\gamma}} - \frac{\partial A_\gamma^{(1)}}{\partial \xi^{(1)\beta}}. \quad (7)$$

If this tensor is nonsingular, the iterative process stops and the Dirac's brackets among the phase space variables are obtained from the inverse matrix $(f_{\gamma\beta}^{(1)})^{-1}$ and, consequently, the Hamilton

equation of motion can be computed and solved as well, as well discussed in [20]. It is well known that a physical system can be described in terms of a symplectic manifold M , classically at least. From a physical point of view, M is the phase space of the system while a nondegenerate closed 2-form f can be identified as being the Poisson bracket. The dynamics of the system is determined just specifying a real-valued function (Hamiltonian) H on phase space, *i.e.*, one these real-valued function solves the Hamilton equation, namely,

$$\iota(X)f = dH, \quad (8)$$

and the classical dynamical trajectories of the system in phase space are obtained. It is important to mention that if f is nondegenerate, Eq.(8) has an unique solution. The nondegeneracy of f means that the linear map $\flat : TM \rightarrow T^*M$ defined by $\flat(X) := \flat(X)f$ is an isomorphism, due to this, the Eq.(8) is solved uniquely for any Hamiltonian ($X = \flat^{-1}(dH)$). On the contrary, the tensor has a zero-mode and a new constraint arises, indicating that the iterative process goes on until the symplectic matrix becomes nonsingular or singular. If this matrix is nonsingular, the Dirac's brackets will be determined. In Ref.[20], the authors consider in detail the case when f is degenerate, which usually arises when constraints are presented on the system. In which case, (M, f) is called presymplectic manifold. As a consequence, the Hamilton equation, Eq.(8), may or may not possess solutions, or possess nonunique solutions. Oppositely, if this matrix is singular and the respective zero-mode does not generate a new constraint, the system has a symmetry.

The systematization of the symplectic embedding formalism begins by assuming that the gauge invariant version of the general Lagrangian ($\tilde{\mathcal{L}}(a_i, \dot{a}_i, t)$) is given by

$$\tilde{\mathcal{L}}(a_i, \dot{a}_i, \varphi_p, t) = \mathcal{L}(a_i, \dot{a}_i, t) + \mathcal{L}_{WZ}(a_i, \dot{a}_i, \varphi_p), \quad (p = 1, 2), \quad (9)$$

where $\varphi_p = (\theta, \dot{\theta})$ and the extra term (\mathcal{L}_{WZ}) depends on the original (a_i, \dot{a}_i) and WZ (φ_p) configuration variables. Indeed, this WZ Lagrangian can be expressed as an expansion in orders of the WZ variable (φ_p) such as

$$\mathcal{L}_{WZ}(a_i, \dot{a}_i, \varphi_p) = \sum_{n=1}^{\infty} v^{(n)}(a_i, \dot{a}_i, \varphi_p), \quad \text{with } v^{(n)}(\varphi_p) \sim \varphi_p^n, \quad (10)$$

which satisfies the following boundary condition,

$$\mathcal{L}_{WZ}(\varphi_p = 0) = 0. \quad (11)$$

The reduction of the Lagrangian, Eq.(9), into its first order form precedes the beginning of conversion process, thus

$$\tilde{\mathcal{L}}^{(0)} = A_{\alpha}^{(0)} \dot{\xi}^{(0)\alpha} + \pi_{\theta} \dot{\theta} - \tilde{V}^{(0)}, \quad (12)$$

where π_θ is the canonical momentum conjugated to the WZ variable, that is,

$$\pi_\theta = \frac{\partial \mathcal{L}_{WZ}}{\partial \dot{\theta}} = \sum_{n=1}^{\infty} \frac{\partial v^{(n)}(a_i, \dot{a}_i, \varphi_p)}{\partial \dot{\theta}}. \quad (13)$$

The expanded symplectic variables are $\tilde{\xi}^{(0)\tilde{\alpha}} \equiv (a_i, p_i, \varphi_p)$ and the new symplectic potential becomes

$$\tilde{V}^{(0)} = V^{(0)} + G(a_i, p_i, \lambda_p), \quad (p = 1, 2), \quad (14)$$

where $\lambda_p = (\theta, \pi_\theta)$. The arbitrary function $G(a_i, p_i, \lambda_p)$ is expressed as an expansion in terms of the WZ fields, namely

$$G(a_i, p_i, \lambda_p) = \sum_{n=0}^{\infty} \mathcal{G}^{(n)}(a_i, p_i, \lambda_p), \quad (15)$$

with

$$\mathcal{G}^{(n)}(a_i, p_i, \lambda_p) \sim \lambda_p^n. \quad (16)$$

In this context, the zeroth one-form canonical momenta are given by

$$\tilde{A}_{\tilde{\alpha}}^{(0)} = \begin{cases} A_{\alpha}^{(0)}, & \text{with } \tilde{\alpha} = 1, 2, \dots, N, \\ \pi_\theta, & \text{with } \tilde{\alpha} = N + 1, \\ 0, & \text{with } \tilde{\alpha} = N + 2. \end{cases} \quad (17)$$

The corresponding symplectic tensor, obtained from the following general relation

$$\tilde{f}_{\tilde{\alpha}\tilde{\beta}}^{(0)} = \frac{\partial \tilde{A}_{\tilde{\beta}}^{(0)}}{\partial \tilde{\xi}^{(0)\tilde{\alpha}}} - \frac{\partial \tilde{A}_{\tilde{\alpha}}^{(0)}}{\partial \tilde{\xi}^{(0)\tilde{\beta}}}, \quad (18)$$

is

$$\tilde{f}_{\tilde{\alpha}\tilde{\beta}}^{(0)} = \begin{pmatrix} f_{\alpha\beta}^{(0)} & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (19)$$

which should be a singular matrix.

The implementation of the symplectic embedding scheme consists in computing the arbitrary function $(G(a_i, p_i, \lambda_p))$. To this end, the correction terms in order of λ_p , within by $\mathcal{G}^{(n)}(a_i, p_i, \lambda_p)$, must be computed as well. If the symplectic matrix, Eq.(19), is singular, it has a zero-mode $\tilde{\varrho}$ and, consequently, we have

$$\tilde{\varrho}^{(0)\tilde{\alpha}} \tilde{f}_{\tilde{\alpha}\tilde{\beta}}^{(0)} = 0, \quad (20)$$

where we assume that this zero-mode is

$$\tilde{\varrho}^{(0)} = (\gamma^\alpha \quad 0 \quad 0), \quad (21)$$

where γ^α , is a generic line matrix. Using the relation given in Eq.(20) together with Eq.(19) and Eq.(21), we get

$$\gamma^\alpha f_{\alpha\beta}^{(0)} = 0. \quad (22)$$

In this way, a zero-mode is obtained and, in agreement with the symplectic formalism, this zero-mode must be contracted with the gradient of the symplectic potential, namely,

$$\tilde{\varrho}^{(0)\tilde{\alpha}} \frac{\partial \tilde{V}^{(0)}}{\partial \xi^{(0)\tilde{\alpha}}} = 0. \quad (23)$$

As a consequence, a constraint arise as being

$$\Omega = \gamma^\alpha \left[\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right]. \quad (24)$$

Due to this, the first-order Lagrangian is rewritten as

$$\tilde{\mathcal{L}}^{(1)} = A_\alpha^{(0)} \dot{\xi}^{(0)\alpha} + \pi_\theta \dot{\theta} + \Omega \dot{\eta} - \tilde{V}^{(1)}, \quad (25)$$

where $\tilde{V}^{(1)} = V^{(0)}$. Note that the symplectic variables are now $\tilde{\xi}^{(1)\tilde{\alpha}} \equiv (a_i, p_i, \eta, \lambda_p)$ (with $\tilde{\alpha} = 1, 2, \dots, N+3$) and the corresponding symplectic matrix becomes

$$\tilde{f}_{\tilde{\alpha}\tilde{\beta}}^{(1)} = \begin{pmatrix} f_{\alpha\beta}^{(0)} & f_{\alpha\eta} & 0 & 0 \\ f_{\eta\beta} & 0 & f_{\eta\theta} & f_{\eta\pi_\theta} \\ 0 & f_{\theta\eta} & 0 & -1 \\ 0 & f_{\pi_\theta\eta} & 1 & 0 \end{pmatrix}, \quad (26)$$

where

$$\begin{aligned} f_{\eta\theta} &= -\frac{\partial}{\partial \theta} \left[\gamma^\alpha \left(\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right) \right], \\ f_{\eta\pi_\theta} &= -\frac{\partial}{\partial \pi_\theta} \left[\gamma^\alpha \left(\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right) \right], \\ f_{\alpha\eta} &= \frac{\partial \Omega}{\partial \xi^{(0)\alpha}} = \frac{\partial}{\partial \xi^{(0)\alpha}} \left[\gamma^\alpha \left(\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right) \right]. \end{aligned} \quad (27)$$

Since our goal is to unveil a WZ symmetry, this symplectic tensor must be singular, consequently, it has a zero-mode, namely,

$$\tilde{\nu}_{(\nu)(a)}^{(1)} = (\mu_{(\nu)}^\alpha \quad 1 \quad a \quad b), \quad (28)$$

which satisfies the relation

$$\tilde{\nu}_{(\nu)(a)}^{(1)\tilde{\alpha}} \tilde{f}_{\tilde{\alpha}\tilde{\beta}}^{(1)} = 0. \quad (29)$$

Note that the parameters (a, b) can be 0 or 1 and ν indicates the number of choices for $\tilde{\nu}^{(1)\tilde{\alpha}}$ [26].

As a consequence, there are two independent set of zero-modes, given by

$$\begin{aligned} \tilde{\nu}_{(\nu)(0)}^{(1)} &= (\mu_{(\nu)}^{\alpha} \quad 1 \quad 0 \quad 1), \\ \tilde{\nu}_{(\nu)(1)}^{(1)} &= (\mu_{(\nu)}^{\alpha} \quad 1 \quad 1 \quad 0). \end{aligned} \quad (30)$$

Note that the matrix elements $\mu_{(\nu)}^{\alpha}$ present some arbitrariness which can be fixed in order to disclose a desired WZ gauge symmetry. In addition, in our formalism the zero-mode $\tilde{\nu}_{(\nu)(a)}^{(1)\tilde{\alpha}}$ is the gauge symmetry generator, which allows to display the symmetry from the geometrical point of view. At this point, we call attention upon the fact that this is an important characteristic since it opens up the possibility to disclose the desired hidden gauge symmetry from the noninvariant model. Different choices of the zero-mode generates different gauge invariant versions of the second class system, however, these gauge invariant descriptions are dynamically equivalent, *i.e.*, there is the possibility to relate this set of independent zero-modes, Eq.(30), through canonical transformation $(\tilde{\nu}_{(\nu)(a)}^{(1)} = T \cdot \tilde{\nu}_{(\nu)(a)}^{(1)})$ where bar means transpose matrix. For example,

$$\begin{pmatrix} \mu_{(\nu)}^{\alpha} \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \mu_{(\nu)}^{\alpha} \\ 1 \\ 1 \\ 0 \end{pmatrix}. \quad (31)$$

While, in the context of the BFFT formalism, different choices for the degenerated matrix X [24] leads to different gauge invariant version of the second class model. It is important to mentioned here that, in Ref.[24], a suitable interpretation and explanation about this result was not given and, also, the author pointed out that not all solutions of the first step of the BFFT method can lead to a solution in the second one, which jeopardize the BFFT embedding process. From the symplectic embedding formalism, this kind of problem can be clarified and understood as well: (i) first, some choices for the degenerated matrix X lead to different gauge invariant version of the second class model, however, they are dynamically equivalent, as shown by the symplectic formalism; (ii) second, some choices for the degenerated matrix X can generate solutions in the first step of the BFFT method that can not lead to a pleasant solution in the second one, which hazards this WZ embedding process. This is interpreted by the symplectic point of view as been the impossibility to introduce some gauge symmetries into the model and, as consequence, an

infinite numbers of WZ counter-terms in Hamiltonian[24] are required. This also happens in the iterative constraint conversion[11], since there is an arbitrariness to change the second class nature of the constraints in first one. Now, it becomes clear that the arbitrariness presents on the BFFT and iterative constraint conversions methods has its origin on the choice of the zero-mode, which generates the desired WZ gauge symmetry.

From relation, Eq.(29), together with Eq.(26) and Eq.(28), some differential equations involving $G(a_i, p_i, \lambda_p)$ are obtained, namely,

$$\begin{aligned} 0 &= \mu_{(\nu)}^\alpha f_{\alpha\beta}^{(0)} + f_{\eta\beta}, \\ 0 &= \mu_{(\nu)}^\alpha f_{\alpha\eta}^{(0)} + a f_{\theta\eta} + b f_{\pi_\theta\eta}, \\ 0 &= f_{\eta\theta}^{(0)} + b, \\ 0 &= f_{\eta\pi_\theta}^{(0)} - a \end{aligned} \quad (32)$$

Solving the relations above, some correction terms, within $\sum_{m=0}^\infty \mathcal{G}^{(m)}(a_i, p_i, \lambda_p)$, can be determined, also including the boundary conditions ($\mathcal{G}^{(0)}(a_i, p_i, \lambda_p = 0)$).

In order to compute the remaining corrections terms of $G(a_i, p_i, \lambda_p)$, we impose that no more constraints arise from the contraction of the zero-mode ($\tilde{\nu}_{(\nu)(a)}^{(1)\tilde{\alpha}}$) with the gradient of potential $\tilde{V}^{(1)}(a_i, p_i, \lambda_p)$. This condition generates a general differential equation, which reads as

$$\begin{aligned} 0 &= \tilde{\nu}_{(\nu)(a)}^{(1)\tilde{\alpha}} \frac{\partial \tilde{V}^{(1)}(a_i, p_i, \lambda_p)}{\partial \tilde{\xi}^{(1)\tilde{\alpha}}} \\ &= \mu_{(\nu)}^\alpha \left[\frac{\partial V^{(1)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} + \frac{\partial G(a_i, p_i, \theta, \pi_\theta)}{\partial \xi^{(1)\alpha}} \right] + a \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \pi_\theta} \\ &= \mu_{(\nu)}^\alpha \left[\frac{\partial V^{(1)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} + \sum_{m=0}^\infty \frac{\partial \mathcal{G}^{(m)}(a_i, p_i, \lambda_p)}{\partial \xi^{(1)\alpha}} \right] + a \sum_{n=0}^\infty \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \theta} \\ &\quad + b \sum_{m=0}^\infty \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}. \end{aligned} \quad (33)$$

The last relation allows us to compute all correction terms in order of λ_p , within $\mathcal{G}^{(n)}(a_i, p_i, \lambda_p)$. Note that this polynomial expansion in terms of λ_p is equal to zero, subsequently, all the coefficients for each order in this WZ variables must be identically null. In view of this, each correction term in orders of λ_p can be determined as well. For a linear correction term, we have

$$0 = \mu_{(\nu)}^\alpha \left[\frac{\partial V^{(0)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} + \frac{\partial \mathcal{G}^{(0)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} \right] + a \frac{\partial \mathcal{G}^{(1)}(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial \mathcal{G}^{(1)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}, \quad (34)$$

where the relation $V^{(1)} = V^{(0)}$ was used. For a quadratic correction term, we get

$$0 = \mu_{(\nu)}^\alpha \left[\frac{\partial \mathcal{G}^{(1)}(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right] + a \frac{\partial \mathcal{G}^{(2)}(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial \mathcal{G}^{(2)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}. \quad (35)$$

From these equations, a recursive equation for $n \geq 2$ is proposed as

$$0 = \mu_{(\nu)}^\alpha \left[\frac{\partial \mathcal{G}^{(n-1)}(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right] + a \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}, \quad (36)$$

which allows us to compute the remaining correction terms in order of θ and π_θ . This iterative process is successively repeated up to Eq.(33) becomes identically null. Then, the new symplectic potential is written as

$$\tilde{V}^{(1)}(a_i, p_i, \lambda_p) = V^{(0)}(a_i, p_i) + G(a_i, p_i, \lambda_p). \quad (37)$$

Due to this, the gauge invariant Hamiltonian is obtained explicitly and the zero-mode $\tilde{\nu}_{(\nu)(a)}^{(1)\tilde{\alpha}}$ is identified as being the generator of the infinitesimal gauge transformation, given by

$$\delta \tilde{\xi}_{(\nu)(a)}^{\tilde{\alpha}} = \varepsilon \tilde{\nu}_{(\nu)(a)}^{(1)\tilde{\alpha}}, \quad (38)$$

where ε is an infinitesimal parameter.

III. REALIZATION OF THE FADDEEVIAN REGULARIZATION IN THE CSM

In Ref.[3] the authors showed that the χQED_2 can be quantized in a consistent unitary way just including the bosonization ambiguity parameter satisfying the condition $a \geq 1$ to avoid tachyonic excitations. Afterwards, Rajaraman[4] studied the canonical structure of the model and showed that there are two cases $a > 1$ and $a = 1$ belonged to distinct classes: the $a = 1$ case presenting four second class constraints belongs to an unambiguous class containing only one representative, while the $a > 1$ case, presenting only two second class constraints, represents a continuous one-parameter class. Due to the distinct constraint structures, the $a > 1$ class presents both massless excitation and massive scalar excitation ($m^2 = \frac{e^2 a^2}{a-1}$), while in the other case there is only massless excitation. It occurs because the chiral Schwinger model with the familiar regularization $a > 1$ has more physical degrees of freedom than it would have were it gauge invariant. However, it is not match with the Faddeev's case[14] whose the commutator between the Gauss law constraint is non-zero. Here, the second class nature of the set of constraints is due to the Poisson bracket of π_0 (canonical momentum conjugated to the scalar potential A_0) and G (the Gauss law) becomes non-zero.

In [13] the author showed that the Poisson bracket involving the Gauss law constraint is non-zero, indeed exhibits the Faddeev's anomaly. In views of this, the author concluded that the

Faddeevian regularization not belong to the class of the usual regularizations. In this new scenario, the gauge field is once again a massive excitation, but the massless fermion that is present has, unlike the usual case, a definite chirality opposite to that entering the interaction with the electromagnetic field. In this work, Mitra showed that with an appropriated choice of the regularization mass term it is possible to close the second class algebra with only three second class constraints. Although this model is not manifestly Lorentz invariant, the Poincaré generators have been constructed [13] and shown to close the relativistic algebra on-shell. The main feature of this new regularization is the presence of a Schwinger term in the Poisson bracket algebra of the Gauss law, which limits the set to only three second class constraints. To see this we start with the CSM Lagrangian with Faddeevian regularization proposed in [15], reads as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + q (g^{\mu\nu} + b \epsilon^{\mu\nu}) \partial_\mu \phi A_\nu + \frac{1}{2} q^2 A_\mu M^{\mu\nu} A_\nu, \quad (39)$$

where the Mitra's regulator was properly generalized. Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $g^{\mu\nu} = \text{diag}(+1, -1)$ and $\epsilon^{01} = -\epsilon^{10} = \epsilon_{10} = 1$. b is a chirality parameter, which can assume the values $b = \pm 1$. The mass-term matrix $M_{\mu\nu}$ is defined as

$$M^{\mu\nu} = \begin{pmatrix} 1 & \alpha \\ \alpha & \beta \end{pmatrix}. \quad (40)$$

To resemble the Rajaraman's $a = 1$ class it was chosen unity coefficient for A_0^2 term. The Rajaraman's class is a singular case in the *space of parameters* since its canonical description has the maximum number of constraints with no massive excitation. This case is reproduced in Eq.(39) if $\alpha = 0$ and $\beta = -1$ in Eq.(40). However, a new class appears if we assume a nonvanishing value for α . For example, with Mitra's choice, $\alpha = -1$ and $\beta = -3$, the photon once again becomes massive ($m^2 = 4q^2$), but the remaining massless fermion has a definite chirality, opposite to that entering the interaction with the electromagnetic field. Although this particular choice is too restrictive, another choices are also possible, that leads, eventually, to a new and interesting consequences. In this work the coefficients α and β are arbitrary *ab initio*, but the mass spectrum will impose a constraint between them. This is verified using the symplectic method[17, 18].

Afterhere, the symplectic method will be used to quantize the original second class model, obtaining the Dirac's brackets and the respective reduced Hamiltonian as well. In order to implement the symplectic method, the original second-order Lagrangian in the velocity, given in Eq.(39), is reduced into its first-order, namely,

$$\mathcal{L}^{(0)} = \pi_\phi \dot{\phi} + \pi^1 \dot{A}_1 - U^{(0)}, \quad (41)$$

where the symplectic potential $U^{(0)}$ is

$$\begin{aligned} U^{(0)} &= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_0(\pi_1' + q^2(\alpha - b)A_1 + q\pi_\phi - qb\phi') \\ &\quad - A_1 \left(qb\pi_\phi + \frac{1}{2}q^2(\beta - b^2)A_1 - q\phi' \right), \end{aligned} \quad (42)$$

where prime represents spatial derivative.

The zeroth-iterative symplectic tensor is given by

$$f^{(0)}(x, y) = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \delta^2(x - y). \quad (43)$$

This matrix is obviously singular, thus, it has the following zero-mode,

$$\nu^{(0)} = (0 \quad 0 \quad 1 \quad 0 \quad 0), \quad (44)$$

that when contracted with the gradient of the potential $U^{(0)}$ generates a constraint, given by,

$$\begin{aligned} \Omega_1 &= \int \nu_\alpha^{(0)}(x) \frac{\partial U^{(0)}(y)}{\partial \xi_\alpha^{(0)}(x)} dy \\ &= \pi_1' + q^2(\alpha - b)A_1 + q\pi_\phi - qb\phi', \end{aligned} \quad (45)$$

that is identified as being the Gauss law, which satisfies the following Poisson algebra,

$$\{\Omega_1(x), \Omega_1(y)\} = -2q^2\alpha\partial_x\delta^2(x - y), \quad (46)$$

where ∂_x represents $\frac{\partial}{\partial x}$. The corresponding bracket in the familiar regularization scheme[3] is zero. That is why the Mitra's model is the one which is in accordance with the Faddeev's scenario[14] in which the Gauss law commutator has a chiral anomaly.

Bringing back the constraint Ω_1 into the canonical sector of the first-order Lagrangian by means of a Lagrange multiplier η , we get the first-iterative Lagrangian $\mathcal{L}^{(1)}$, reads as

$$\mathcal{L}^{(1)} = \pi_\phi \dot{\phi} + \pi^1 \dot{A}_1 + \Omega_1 \dot{\eta} - U^{(1)}, \quad (47)$$

with the first-order symplectic potential,

$$U^{(1)} = \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_1 \left(qb\pi_\phi + \frac{1}{2}q^2(\beta - b^2)A_1 - q\phi' \right). \quad (48)$$

The corresponding matrix $f^{(1)}(x, y)$ is then[27]

$$f^{(1)}(x, y) = \begin{pmatrix} 0 & -1 & 0 & 0 & -qb\partial_y \\ 1 & 0 & 0 & 0 & q \\ 0 & 0 & 0 & -1 & q^2\sigma \\ 0 & 0 & 1 & 0 & \partial_y \\ qb\partial_x & -q & -q^2\sigma & -\partial_x & 0 \end{pmatrix} \delta^2(x - y), \quad (49)$$

that is singular and has a zero-mode, given by,

$$\nu^{(1)} = (-q \quad -qb\partial_x \quad -\partial_x \quad q^2\sigma \quad 1), \quad (50)$$

that generates the following constraint,

$$\begin{aligned} \Omega_2 &= \int \nu_\alpha^{(1)}(x) \frac{\partial U^{(1)}(y)}{\partial \xi_\alpha^{(1)}(x)} dy \\ &= q^2\sigma\pi_1 + q^2(\beta + 1)A_1'. \end{aligned} \quad (51)$$

The twice-iterated Lagrangian, obtained after including the constraint, given in Eq.(51), into Lagrangian, Eq.(47), by means of a Lagrange multiplier ζ , reads

$$\mathcal{L}^{(2)} = \pi_\phi \dot{\phi} + \pi^1 \dot{A}_1 + \Omega_1 \dot{\eta} + \Omega_2 \dot{\zeta} - U^{(2)}, \quad (52)$$

where $U^{(2)} = U^{(1)}|_{\Omega_2=0}$.

The matrix $f^{(2)}(x, y)$ is[28]

$$f^{(2)}(x, y) = \begin{pmatrix} 0 & -1 & 0 & 0 & -qb\partial_y & 0 \\ 1 & 0 & 0 & 0 & q & 0 \\ 0 & 0 & 0 & -1 & q^2\sigma & q^2\rho\partial_y \\ 0 & 0 & 1 & 0 & \partial_y & q^2\sigma \\ qb\partial_x & -q & -q^2\sigma & -\partial_x & 0 & 0 \\ 0 & 0 & -q^2\rho\partial_x & -q^2\sigma & 0 & 0 \end{pmatrix} \delta(x - y), \quad (53)$$

that is a nonsingular matrix. Then we can identify it as the symplectic tensor of the constrained theory. The inverse of $f^{(2)}(x, y)$ gives, after a straightforward calculation, the Dirac brackets among the physical fields,

$$\begin{aligned} \{\phi(x), \phi(y)\}^* &= -\frac{1}{2\alpha} \Theta(x - y), \\ \{\phi(x), \pi_\phi(y)\}^* &= \frac{(2\alpha - b)}{2\alpha} \delta(x - y), \\ \{\phi(x), A_1(y)\}^* &= -\frac{1}{2q\alpha} \delta(x - y), \\ \{\phi(x), \pi_1(y)\}^* &= \frac{q\sigma}{2\alpha} \Theta(x - y), \\ \{\pi_\phi(x), \pi_\phi(y)\}^* &= \frac{1}{2\alpha} \partial_x \delta(x - y), \\ \{\pi_\phi(x), A_1(y)\}^* &= \frac{b}{2q\alpha} \partial_x \delta(x - y), \\ \{\pi_\phi(x), \pi_1(y)\}^* &= -\frac{qb\sigma}{2\alpha} \delta(x - y), \\ \{A_1(x), A_1(y)\}^* &= \frac{1}{2q^2\alpha} \partial_x \delta(x - y), \\ \{A_1(x), \pi_1(y)\}^* &= \left(\frac{\alpha + b}{2\alpha}\right) \delta(x - y), \\ \{\pi_1(x), \pi_1(y)\}^* &= -\frac{q^2\sigma^2}{2\alpha} \Theta(x - y), \end{aligned} \quad (54)$$

where $\Theta(x - y)$ represents the sign function. This means that the Mitra model is not a gauge invariant theory. The second-iterative symplectic potential $U^{(2)}$ is identified as the reduced Hamiltonian, given by,

$$\begin{aligned} H_r = \int dx \left\{ \frac{1}{2} \pi_1^2 + \alpha \pi_1 A_1' + q(1 - b\alpha) A_1 \phi' + \phi'^2 + \right. \\ \left. + \frac{b}{q} \phi' \pi_1' + \frac{1}{2q^2} \pi_1'^2 + \frac{1}{2} q^2 (\alpha^2 - \beta) A_1^2 \right\}, \end{aligned} \quad (55)$$

where the constraints Ω_1 and Ω_2 were assumed equal to zero in a strong way.

At this stage, we are interested to compute the energy spectrum. To this end, we use the reduced Hamiltonian, Eq.(55), and the Dirac's brackets, Eq.(54), to obtain the following equations of motion for the fields,

$$\begin{aligned}\dot{\phi} &= b\phi' - \frac{1}{q}\pi'_1 + \frac{q}{2\alpha}(1 - 2\alpha^2 + \beta)A_1, \\ \dot{\pi}_1 &= -b\pi'_1 + \frac{q^2}{2\alpha}[(b - \alpha)(1 - \alpha^2) - (b + \alpha)(\alpha^2 - \beta)]A_1, \\ \dot{A}_1 &= \left(\frac{\alpha + b}{2\alpha}\right)\pi_1 - \left(\frac{1 + \beta}{2\alpha}\right)A'_1.\end{aligned}\tag{56}$$

Now, we are ready to determine the spectrum of the model. Isolating π^1 from the constraint Ω_2 , given in Eq.(51), and substituting in Eq.(56), we get

$$\begin{aligned}\left(\frac{2\alpha}{\alpha + b}\right)\ddot{A}_1 + b\left(\frac{1 + \beta}{\alpha + b}\right)A''_1 &= -\left(\frac{2b\alpha}{\alpha + b} + \frac{1 + \beta}{\alpha + b}\right)\dot{A}'_1 + \\ &+ \frac{q^2}{2\alpha}[(b - \alpha)(1 - \alpha^2) - (b + \alpha)(\alpha^2 - \beta)]A_1.\end{aligned}\tag{57}$$

To get a massive Klein-Gordon equation for the photon field we must set

$$(1 + \beta) + b(2\alpha) = 0,\tag{58}$$

which relates α and β and shows that the regularization ambiguity adopted in [13] can be extended to a continuous one-parameter class (for a chosen chirality). We have, using Eq.(57) and Eq.(58), the following mass formula for the massive excitation of the spectrum,

$$m^2 = q^2 \frac{1 + b\alpha}{b\alpha}.\tag{59}$$

Note that to avoid tachyonic excitations, α is further restricted to satisfy $b\alpha = |\alpha|$, so $\alpha \rightarrow -\alpha$ interchanges from one chirality to another. Observe that in the limit $\alpha \rightarrow 0$ the massive excitation becomes infinitely heavy and decouples from the spectrum. This leads us back to the four-constraints class. It is interesting to see that the redefinition of the parameter as $a = 1 + |\alpha|$ leads to,

$$m^2 = \frac{q^2 a^2}{a - 1},\tag{60}$$

which is the celebrated mass formula of the chiral Schwinger model, showing that the parameter dependence of the mass spectrum is identical to both the Jackiw-Rajaraman and the Faddeevian regularizations.

Let us next discuss the massless sector of the spectrum. To disclose the presence of the chiral excitation we need to diagonalize the reduced Hamiltonian, Eq.(55). This procedure may, at least in principle, impose further restrictions over α . This all boils down to find the correct linear combination of the fields leading to the free chiral equation of motion. To this end we substitute π^1 from its definition and A_1 from the Klein-Gordon equation into Eq.(56) to obtain

$$0 = \frac{\partial}{\partial t} \left\{ \phi + \frac{q}{2\alpha} \left(\frac{2 + 2b\alpha - \alpha^2}{m^2} \right) \dot{A}_1 + \frac{1}{q} \left(\frac{\alpha}{\alpha + b} \right) A'_1 \right\} - \frac{\partial}{\partial x} \left\{ b\phi - \frac{1}{q} \left(\frac{\alpha}{\alpha + b} \right) \dot{A}_1 + \left[\frac{q}{2\alpha} \left(\frac{2 + 2b\alpha - \alpha^2}{m^2} \right) - \frac{1}{q} \left(\frac{2b\alpha}{\alpha + b} \right) \right] \right\}. \quad (61)$$

This expression becomes the equation of motion for a self-dual boson χ , given by,

$$\dot{\chi} - b\chi' = 0, \quad (62)$$

if we identify the coefficients for \dot{A}_1 and A'_1 in the two independent terms of Eq.(61) with,

$$\chi = \phi + \frac{1}{q} \left(\frac{\alpha}{\alpha + b} \right) (A'_1 - b\dot{A}_1). \quad (63)$$

This field redefinition, differently from the case of the massive field whose construction imposed condition, Eq.(58), does not restrain the parameter α any further. Using the constraints Ω_1 and Ω_2 , given in Eq.(45) and Eq.(51) respectively, and Eq.(62), all the fields can be expressed as functions of the free massive scalar A_1 and the free chiral boson χ , interpreted as the bosonized Weyl fermion. The main result of this section is now complete, i.e., the construction of the one-parameter class regularization generalizing Mitra's proposal. In the next section, this general model will be reformulated as a gauge theory in order to eliminate the chiral anomaly.

IV. EMBEDDING THE CHIRAL SCHWINGER MODEL WITH THE FADDEEVIAN REGULARIZATION

In order to begin with the WZ embedding formulation, some WZ counter-terms, embraced by \mathcal{L}_{WZ} , are introduced into the original Lagrangian, leading to the gauge invariant Lagrangian, namely,

$$\tilde{\mathcal{L}} = \mathcal{L} + \mathcal{L}_{WZ} \quad (64)$$

In agreement with the symplectic embedding formalism, the invariant Lagrangian above, Eq.(64), must be reduced into its first-order form, given by

$$\tilde{\mathcal{L}}^{(0)} = \pi_\phi \dot{\phi} + \pi^1 \dot{A}_1 + \pi_\theta \dot{\theta} - \tilde{U}^{(0)}, \quad (65)$$

where

$$\begin{aligned} \tilde{U}^{(0)} &= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_0(\pi_1' + q^2(\alpha - b)A_1 + q\pi_\phi - qb\phi') \\ &- A_1 \left(qb\pi_\phi + \frac{1}{2}q^2(\beta - b^2)A_1 - q\phi' \right) + G(\phi, \pi_\phi, A_0, A_1, \pi_1, \lambda_p), \end{aligned} \quad (66)$$

where the arbitrary function G is

$$G(\phi, \pi_\phi, A_0, A_1, \pi_1, \lambda_p) = \sum_{n=0}^{\infty} \mathcal{G}^{(n)}(\phi, \pi_\phi, A_0, A_1, \pi_1, \lambda_p), \quad (67)$$

where the function expanded in terms the WZ variables ($\lambda_p = (\theta, \pi_\theta)$) is given by

$$\mathcal{G}^{(n)} \sim (\lambda_p)^n. \quad (68)$$

The symplectic variables are given by $\tilde{\xi}_\alpha^{(0)} = (\phi, \pi_\phi, A_0, A_1, \pi_1, \lambda_p)$. The corresponding symplectic matrix $\tilde{f}^{(0)}(x, y)$ reads

$$\tilde{f}^{(0)}(x, y) = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \delta^{(2)}(x - y). \quad (69)$$

As the matrix is singular, it has a zero-mode,

$$\tilde{\nu}^{(0)} = (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) \quad (70)$$

which generates the following constraint

$$\Omega = -\pi'_1 - q^2(\alpha - b)A_1 - q\pi_\phi + qb\phi' + \int d\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \quad (71)$$

after the contraction with the gradient of the symplectic potential.

Following the symplectic embedding formalism, this constraint is introduced into the kinetical sector of the zeroth-iterative Lagrangian, Eq.(65), through a Lagrange multiplier (η) , which leads to the first-iterative Lagrangian, given by

$$\tilde{\mathcal{L}}^{(1)} = \pi_\phi \dot{\phi} + \pi_1 \dot{A}^1 + \pi_\theta \dot{\theta} + \Omega \dot{\eta} - \tilde{U}^{(1)}, \quad (72)$$

where $\tilde{U}^{(1)} = \tilde{U}^{(0)}|_{\Omega=0}$. Now, the symplectic variables are $\tilde{\xi}_{\tilde{\alpha}}^{(1)} = (\phi, \pi_\phi, A_0, A_1, \pi_1, \eta, \lambda_p)$ with the respective symplectic matrix,

$$\tilde{f}^{(1)} = \begin{pmatrix} 0 & -\delta^{(2)}(x-y) & 0 & 0 & 0 & f_{\phi\eta}^{(1)} & 0 & 0 \\ \delta^{(2)}(x-y) & 0 & 0 & 0 & 0 & f_{\pi_\phi\eta}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_{A_0\eta}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta^{(2)}(x-y) & f_{A_1\eta}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & \delta^{(2)}(x-y) & 0 & f_{\pi_1\eta}^{(1)} & 0 & 0 \\ f_{\eta\phi}^{(1)} & f_{\eta\pi_\phi}^{(1)} & f_{\eta A_0}^{(1)} & f_{\eta A_1}^{(1)} & f_{\eta\pi_1}^{(1)} & 0 & f_{\eta\theta}^{(1)} & f_{\eta\pi_\theta}^{(1)} \\ 0 & 0 & 0 & 0 & 0 & f_{\theta\eta}^{(1)} & 0 & -\delta^{(2)}(x-y) \\ 0 & 0 & 0 & 0 & 0 & f_{\pi_\theta\eta}^{(1)} & \delta^{(2)}(x-y) & 0 \end{pmatrix}. \quad (73)$$

with

$$\begin{aligned} f_{\phi\eta}^{(1)} &= qb\partial_y\delta^{(2)}(x-y) + \frac{\delta}{\delta\phi(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\ f_{\pi_\phi\eta}^{(1)} &= -q\delta^{(2)}(x-y) + \frac{\delta}{\delta\pi_\phi(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\ f_{A_0\eta}^{(1)} &= \frac{\delta}{\delta A_0(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\ f_{A_1\eta}^{(1)} &= -q^2(\alpha-b)\delta^{(2)}(x-y) + \frac{\delta}{\delta A_1(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\ f_{\pi_1\eta}^{(1)} &= -\partial_y\delta^{(2)}(x-y) + \frac{\delta}{\delta\pi_1(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \end{aligned} \quad (74)$$

$$f_{\theta\eta}^{(1)} = \frac{\delta}{\delta\theta(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(y)},$$

$$f_{\pi\theta\eta}^{(1)} = \frac{\delta}{\delta\pi_\theta(x)} \int d\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(y)},$$

Now, in order to unveil the gauge symmetry and to put our result in perspective with others, we make a “educated guess” for the zero-mode, namely,

$$\tilde{\nu}^{(1)\widetilde{\alpha}} = (q \quad qb\partial_x \quad 0 \quad \partial_x \quad 0 \quad 1 \quad \Delta \quad -q^2c\partial_x/\Delta), \quad (75)$$

where $\Delta^2 = q^2(\beta+1)$ and c is a chiral parameter in the WZ sector. The contraction of this zero-mode with the symplectic tensor leads to eight differential equations, which are written as

$$\begin{aligned} 0 &= \int dx \frac{\delta}{\delta\phi(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\ 0 &= \int dx \frac{\delta}{\delta\pi_\phi(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\ 0 &= \int dx \frac{\delta}{\delta A_0(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\ 0 &= \int dx \left[q^2(\alpha - b)\delta^{(2)}(x - y) - \frac{\delta}{\delta A_1(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right], \\ 0 &= \int dx \frac{\delta}{\delta\pi_1(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\ 0 &= \int dx \left[-q^2(\alpha + b)\partial_x\delta^{(2)}(x - y) + \partial_x \frac{\delta}{\delta A_1(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} + \right. \\ &\quad \left. + \Delta \frac{\delta}{\delta\theta(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} - \frac{q^2}{\Delta} c\partial_x \frac{\delta}{\delta\pi_\theta(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right], \\ 0 &= \int dx \left[-\frac{q^2}{\Delta} c\partial_x\delta^{(2)}(x - y) - \frac{\delta}{\delta\theta(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right], \\ 0 &= \int dx \left[-\Delta\delta^{(2)}(x - y) - \frac{\delta}{\delta\pi_\theta(y)} \int_{\omega} \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right]. \end{aligned} \quad (76)$$

From the fourth relation above, we get the boundary condition, which is written as

$$\mathcal{G}^{(0)} = q^2(\alpha - b)A_0A_1. \quad (77)$$

So, the zeroth correction term is

$$\mathcal{G}^{(0)} = q^2(\alpha - b)A_0A_1 + \mathcal{G}^{(0)}(A_1), \quad (78)$$

since from the third relation above, we see that $\mathcal{G}^{(0)}$ has no quadratic dependence in terms of A_0 . While from the seventh and eighth relation above, the first correction term, at least, is obtained partially as being

$$\mathcal{G}^{(1)} = -\frac{q^2}{\Delta} c \partial_1 \theta A_0 - \Delta \pi_\theta A_0. \quad (79)$$

It is important to notice that the relation above can not envelop all of the first-correction terms, because some of them can not depend on the temporal component of the potential field. In view of this, we rewrite this term as

$$\mathcal{G}^{(1)} = -\frac{q^2}{\Delta} c \partial_1 \theta A_0 - \Delta \pi_\theta A_0 + \mathcal{G}^{(1)}(A_1, \lambda_p). \quad (80)$$

Thus, the constraint, given in Eq. (71), becomes

$$\Omega = -\pi'_1 - q\pi_\phi + qb\phi' - \frac{q^2}{\Delta} c\theta' - \Delta\pi_\theta. \quad (81)$$

This constraint satisfies the following Poisson algebra

$$\{\Omega(x), \Omega(y)\} = 2q^2(b-c)\partial_y\delta(x-y). \quad (82)$$

Note that the elimination of the chiral anomaly above imposes a condition on the chiral parameters present on the original model and on the WZ sector, b and c , respectively, *i.e.*, $b = c$. Hence, there is a specific set of WZ gauge symmetries that can deal with the chiral anomaly.

The symplectic potential can be expressed as

$$\begin{aligned} \tilde{U}^{(1)} &= \tilde{U}^{(0)}|_{\Omega=0} \\ &= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_1 \left(qb\pi_\phi + \frac{1}{2}q^2(\beta - b^2)A_1 - q\phi' \right) \\ &\quad + \sum_{n=1}^{\infty} \mathcal{G}^{(n)}(A_1, \lambda_p) + \mathcal{G}^{(0)}(A_1). \end{aligned} \quad (83)$$

Now, it becomes necessary to guarantee that no more constraint arises. To this end, we impose that the contraction of the zero-mode, Eq.(75), with the gradient of the symplectic potential, Eq.(83), does not produce a new constraint, namely,

$$0 = \int \tilde{\nu}^{(1)\tilde{\alpha}} \frac{\partial U^{(1)}(y)}{\partial \tilde{\xi}^{(1)\tilde{\alpha}}(x)} dy,$$

$$\begin{aligned}
&= \int dy \left[-q^2(\beta+1)A_1(y)\partial_y\delta^{(2)}(x-y) + \sum_{n=1}^{\infty} \partial_x \frac{\delta\mathcal{G}^{(n)}(A_1, \lambda_p)(y)}{\delta A_1(x)} + \right. \\
&\quad \left. + \partial_x \frac{\delta\mathcal{G}^{(0)}(A_1)(y)}{\delta A_1(x)} + \Delta \sum_{n=1}^{\infty} \frac{\delta\mathcal{G}^{(n)}(A_1, \lambda_p)(y)}{\delta\theta(x)} - \frac{q^2}{\Delta} b \partial_x \sum_{n=1}^{\infty} \frac{\delta\mathcal{G}^{(n)}(A_1, \lambda_p)(y)}{\delta\pi_\theta(x)} \right]. \quad (84)
\end{aligned}$$

Which is a polynomial expression in order of the WZ fields (λ_p) . For the zeroth relation written in terms of WZ fields, we get

$$\begin{aligned}
0 &= \int dy \left[-q^2(\beta+1)A_1(y)\partial_y\delta^{(2)}(x-y) + \partial_x \frac{\delta\mathcal{G}^{(0)}(A_1)(y)}{\delta A_1(x)} + \right. \\
&\quad \left. + \Delta \frac{\delta\mathcal{G}^{(1)}(A_1, \lambda_p)(y)}{\delta\theta(x)} - \frac{q^2}{\Delta} b \partial_x \frac{\delta\mathcal{G}^{(1)}(A_1, \lambda_p)(y)}{\delta\pi_\theta(x)} \right]. \quad (85)
\end{aligned}$$

At this point, it is important to notice that some degeneracy appears, since we can not solve the relation above leading to an unique result, *i.e.*, this relation has solution but it is not unique. At first, this sound quite bad, however, this shows how powerful the symplectic embedding formalism can be: different choices for $\mathcal{G}^{(0)}(A_1)$ and $\mathcal{G}^{(1)}(A_1, \lambda_p)$ leads to distinct gauge invariant Hamiltonian descriptions for the noninvariant model, but with the same WZ gauge symmetry. It is a new feature in the WZ embedding concept that could be revealed in the symplectic embedding formalism. On the other hand, this relation can lead to a hard computation of the gauge invariant symplectic potential just assuming a bad solution for $\mathcal{G}^{(0)}(A_1)$ and $\mathcal{G}^{(1)}(A_1, \lambda_p)$.

As we are interested in comparing our result with others, we tackle a fine solution which becomes the computation of correction terms in WZ fields an easy task. The chosen solutions for Eq.(85) are

$$\begin{aligned}
\mathcal{G}^{(0)}(A_1) &= -\frac{q^4}{2\Delta^2} A_1^2, \\
\mathcal{G}^{(1)}(A_1, \lambda_p) &= \Delta A_1 \partial_1 \theta - \frac{q^2}{\Delta} b A_1 \pi_\theta. \quad (86)
\end{aligned}$$

So, the symplectic potential becomes

$$\begin{aligned}
\tilde{U}^{(1)} &= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_1 \left(q b \pi_\phi + \frac{1}{2} q^2 (\beta - b^2) A_1 - q \phi' + \right. \\
&\quad \left. + \frac{q^4}{2\Delta^2} b A_1 - \Delta \partial_1 \theta + \frac{q^2}{\Delta} b \pi_\theta \right) + \sum_{n=2}^{\infty} \mathcal{G}^{(n)}(A_1, \lambda_p). \quad (87)
\end{aligned}$$

Again, we use the Eq.(84), which allows us to compute the quadratic correction term in WZ fields,

$$\begin{aligned}
0 &= \int dy \left[\Delta \partial_1 \theta(y) \partial_y \delta^{(2)}(x-y) - \frac{q^2}{\Delta} \pi_\theta(y) \partial_x \delta^2(x-y) + \right. \\
&\quad \left. + \Delta \frac{\delta\mathcal{G}^{(2)}(A_1, \lambda_p)(y)}{\delta\theta(x)} - \frac{q^2}{\Delta} \partial_x \frac{\delta\mathcal{G}^{(2)}(A_1, \lambda_p)(y)}{\delta\pi_\theta(x)} \right], \quad (88)
\end{aligned}$$

which leads to the following solution,

$$\mathcal{G}^{(2)} = \frac{q^2}{2}(\partial_1\theta)^2 - \frac{1}{2}(\pi_\theta)^2. \quad (89)$$

As this last correction term $\mathcal{G}^{(2)}$ has dependence only on the WZ fields, the correction terms $\mathcal{G}^{(n)} = 0$ for $n \geq 3$. Due to this, the symplectic potential, identified as being the gauge invariant Hamiltonian, is

$$\begin{aligned} \tilde{\mathcal{H}} = \tilde{U}^{(1)} &= \frac{1}{2} \left(\pi_1^2 + \pi_\phi^2 + (\partial_1\phi)^2 + (\partial_1\theta)^2 - (\pi_\theta)^2 \right) \\ &+ A_0 \left(-\partial_1\pi_1 - q\pi_\phi + qb\partial_1\phi - \frac{q^2}{\Delta}b\partial_1\theta - \Delta\pi_\theta \right) \\ &+ A_1 \left(-qb\pi_\phi + \frac{q^2}{2} \frac{\beta^2}{(\beta+1)} A_1 + q\partial_1\phi - \frac{q^2}{\Delta}b\pi_\theta + \Delta\partial_1\theta \right), \end{aligned} \quad (90)$$

which is the same result obtained in [11], when $\beta = -a$.

To complete the gauge invariant reformulation of the model, the infinitesimal gauge transformation will be also computed. In agreement with the symplectic method, the zero-mode, $\tilde{\nu}^{(1)}$, Eq.(75), is the generator of the infinitesimal gauge transformation ($\delta\mathcal{O} = \varepsilon\tilde{\nu}$), given by

$$\begin{aligned} \delta\phi &= q\varepsilon, \\ \delta\pi_\phi &= -qb\partial\varepsilon, \\ \delta A_0 &= 0, \\ \delta A_1 &= -\partial\varepsilon, \\ \delta\pi_1 &= 0, \\ \delta\eta &= \varepsilon, \\ \delta\theta &= \Delta\varepsilon, \\ \delta\pi_\theta &= \frac{q^2}{\Delta}b\partial\varepsilon, \end{aligned} \quad (91)$$

where ε is an infinitesimal time dependent parameter.

V. FINAL DISCUSSIONS

In this work the bosonized form of the CSM fermionic determinant parameterized by a single real number, which extends early regularizations [3, 13], was studied from the symplectic point of view. Afterwards, we reproduce the spectrum that has been shown in Ref.[15] to consist of a chiral boson and a massive photon field. The mass formula for the scalar excitation was shown to

reproduce the Jackiw-Rajaraman result, while the massless excitation was shown to reproduce the Mitra result. In view of this, unitarity condition restrains the range of the regularization parameter in a similar way. This noninvariant model was reformulated as a gauge invariant model *via* the symplectic embedding formalism, where we were able to produce the gauge invariant version of the model, in order to eliminate the chiral anomaly in the Gauss law commutator. It is important to mention here that the gauge invariant version of CSM, given in Ref.[11], can be also obtained when $\beta = -a$. Notice that the invariant theory was obtained with the introduction of a finite number of WZ terms. It is also important to emphasize that we can obtain different Hamiltonian formulations for the model. Different choices of the zero-mode generate different gauge invariant versions of the second class system, however, these gauge invariant descriptions are dynamically equivalent, *i.e.*, there is the possibility to relate this set of independent zero-modes through canonical transformation [25]. Another interesting point discussed in this work is the geometrical interpretation given to the degeneracy of the matrix X presents on the BFFT formalism and the arbitrariness on the iterative method. Different choices for this matrix X , or way to turn second class constraints to first one in the iterative method, leads to distinct gauge invariant version of the second class model. But, from the symplectic embedding point of view, these invariant descriptions are equivalent. Besides, it was also possible to reveal an interesting feature in the symplectic embedding formulation: the possibility to introduce boundary conditions that has no dependence on the WZ variables, which opens new *modus* to run the embedding process. This is a new concept which lifts the WZ embedding idea to the next level.

VI. ACKNOWLEDGMENTS

This work is supported in part by FAPEMIG and CNPq, Brazilian Research Agencies. One of us, W. Oliveira, would like to thank Prof. Dr. José Maria Filardo Bassalo, his friend and teacher, for the great stimulation gave in the beginning of his career.

-
- [1] J.Ananias Neto, C.Neves and W.Oliveira, Phys. Rev. D63 (2001) 085018.
 - [2] A.C.R. Mendes, W. Oliveira and C. Neves, Nucl. Phys. B (Proc. Suppl.) v127 (2004) 170.
 - [3] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. 54 (1985) 1219; 54 (1985) 2060(E).
 - [4] R. Rajaraman, Phys. Lett. B154 (1985) 305.
 - [5] R. Banerjee, Phys. Rev. Lett. 56 (1986) 1889.

- S. Miyake and K. Shizuya, Phys. Rev. D 36 (1987) 3781.
- K. Harada and I. Tsutsui, Phys. Lett. B 171 (1987) 311.
- [6] R. Rajarama, Phys. Lett. B 162 (1985) 148.
- [7] J. Lott and R. Rajarama, Phys. Lett. B165 (1985) 321.
- [8] H.D. Girotti, H.J. Rothe and K.D. Rothe, Phys. Rev. D33 (1986) 514.
- [9] N.K. Falk and G. Kramer, Ann. Phys. 176 (1987) 330.
- [10] S. Miyake and K. Shizuya, Phys. Rev. D36 (1987) 3781.
- [11] C. Wotzasek, Int. J. Mod. Phys. A5 (1990) 1123.
- [12] L. Faddeev and S.L. Shatashvilli, Phys. Lett. B167 (1986) 225.
- [13] P. Mitra, Phys. Lett. B 284 (1992) 23.
- S. Mukhopadhyay and P. Mitra, Z. Phys. C67 (1995) 525.
- S. Mukhopadhyay and P. Mitra, Ann. Phys. 241 (1995) 68.
- [14] L.D. Faddeev, Phys. Lett. B145 (1984) 81.
- [15] E.M.C. Abreu, A. Ilha, C. Neves and C. Wotzasek, Phys. Rev. D61 (1999) 25014.
- [16] C. Wotzasek, *Soldering formalism - theory and applications*, hep-th/9806005.
- [17] L.D. Faddeev and R. Jackiw, Phys. Rev. Lett. 60 (1988) 1692.
- N.M.J. Woodhouse, *Geometric Quantization* (Clarendon Press, Oxford, 1980).
- [18] J. Barcelos Neto and C. Wotzasek, Mod. Phys. Lett. A7 (1992) 1172.
- [19] P.A.M. Dirac, Proc. Roy. Soc. A257 (1960) 32; *Lectures on Quantum Mechanics* (Yeshiva University Press, New York, 1964).
- A. Hanson, T. Regge and C. Teitelboim, *Constrained Hamiltonian Systems* (Accademia Nazionale dei Lincei, Roma 1976).
- K. Sundermeyer, *Constrained Dynamics, Lectures Notes in Physics* (Springer, New York, 1982), vol 169.
- [20] M.J. Gotay, J.M. Nester and G. Hinds, J. Math. Phys. 19(11) 2388(1978).
- [21] C. Wotzasek, Phys. Rev. Lett. 66 (1991) 129.
- [22] I.A. Batalin and E.S. Fradkin, Nucl. Phys. B279 (1987) 514.
- I.A. Batalin and I.V. Yutin, Int. J. Mod. Phys. A (1991) 3255.
- [23] M. Moshe and Y. Oz, Phys. Lett. B224 (1989) 145.
- T. Fujiwara, Y. Igarashi and J. Kubo, Nucl. Phys. B341 (1990) 695.
- [24] J. Barcelos-Neto, Phys. Rev. D55 (1997) 2265.
- [25] A.C.R. Mendes, W. Oliveira and C. Neves, J. Phys. A: Math. and Gen. 37 (2004) 1927.
- [26] It is important to notice that ν is not a fixed parameter
- [27] $\sigma = \alpha - b$
- [28] $\varrho = \beta + 1$